Chapter 15

Random Forest

Problem 15.1

$$
\begin{aligned}
\operatorname{Var}\left(\frac{1}{B}\sum_{i=1}^{B}X_i\right) =& \mathbb{E}\left[\left(\frac{1}{B}\sum_{i=1}^{B}X_i\right)^2\right] - \mathbb{E}\left[\frac{1}{B}\sum_{i=1}^{B}X_i\right]^2 \\
=& \frac{1}{B^2}\mathbb{E}\left[\left(\sum_{i=1}^{B}X_i\right)\left(\sum_{i=1}^{B}X_i\right)\right] - \mu^2 \\
=& \frac{1}{B^2}\mathbb{E}\left[\sum_{i=1}^{B}\sum_{j=1}^{B}X_iX_j\right] - \mu^2 \\
=& \frac{1}{B^2}\mathbb{E}\left[\sum_{i=1}^{B}\sum_{j=1}^{B}X_iX_j - B^2\mu^2\right] \\
=& \frac{1}{B^2}\mathbb{E}\left[\sum_{i=1}^{B}\sum_{j=1}^{B}(X_i-\mu)(X_j-\mu)\right] \\
=& \frac{1}{B^2}\mathbb{E}\left[B\sigma^2 + (B^2-B)\rho\sigma^2\right] \\
=& \frac{1-\rho}{B}\sigma^2 + \rho\sigma^2 \to \rho\sigma^2 \text{ as } B \to \infty
\end{aligned}
$$

The result fails if ρ is negative, but the truth is, a native ρ will lead to correlation matrix that is no longer positive definite and thus is not a valid setting.